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UNIVERSIDAD
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DE CHILE

Departamento de Ingeniería de
Transporte y Logística

CENTRE OF
EXCELLENCE

ROUTE BASED EQUILIBRIUM ASSIGNMENT IN CONGESTED NETWORKS

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OUTLINE

- Motivation
- Introduction
- Problem formulation
- Solution algorithm
- Results

MOTIVATION

- De Cea & Fernandez (1993)

    - Implement congestion effect in boarding process
    - Introduce the concept of effective frequencies
    - Only work based on routes
    - Semi-congested models
    - Various approximations

INTRODUCTION

Trip Generation — *How many trips are generated?*

↓

Trip Distribution — *Where do trips go?*

↓

Mode Choice — *What travel mode is used for each trip?*

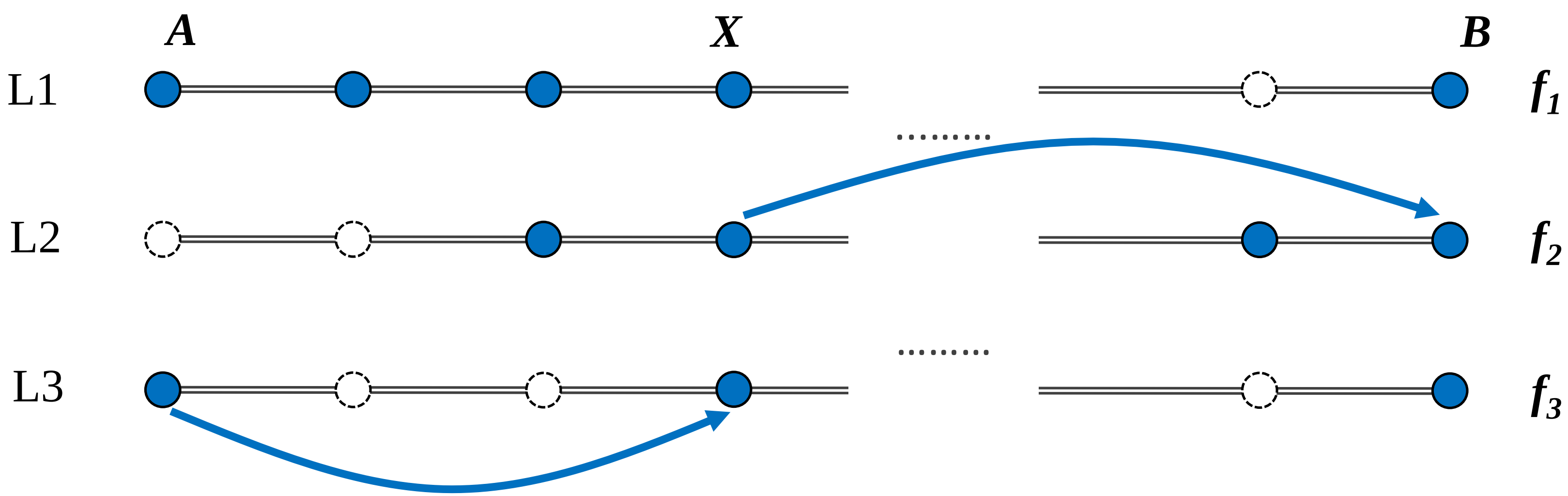
↓

Trip Assignment — *What is the route of each trip?*

- Combinatorial nature of the problem
- Other complexities

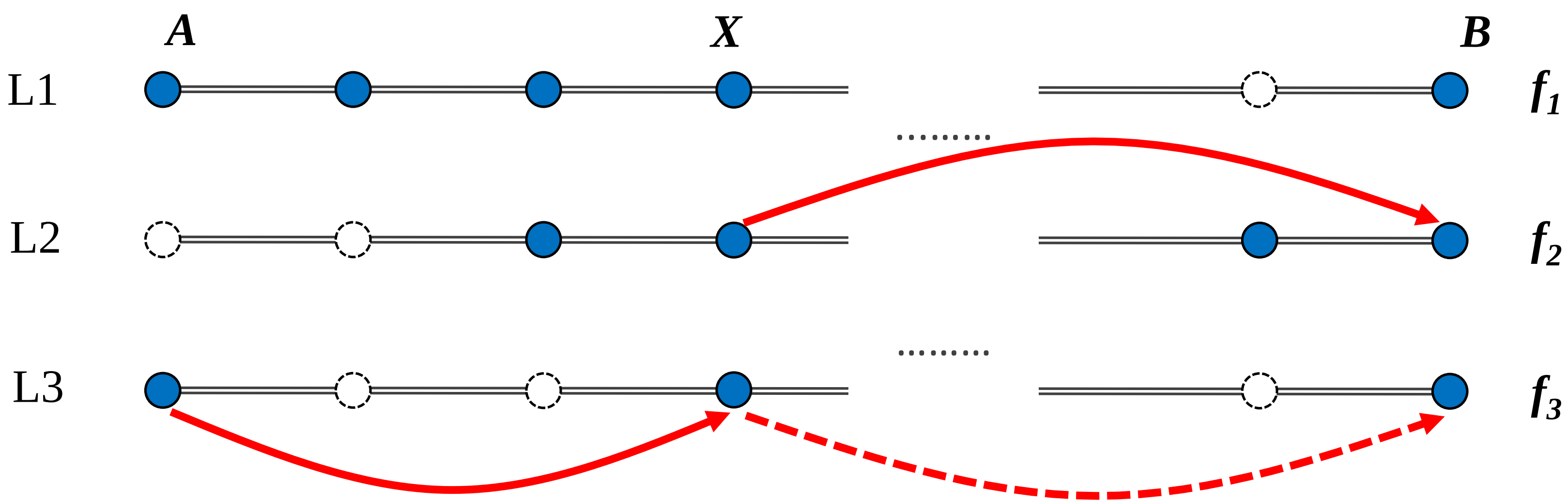
TRIP CHARACTERIZATION

- Itineraries
- Routes
- Hyperpaths (Strategies)



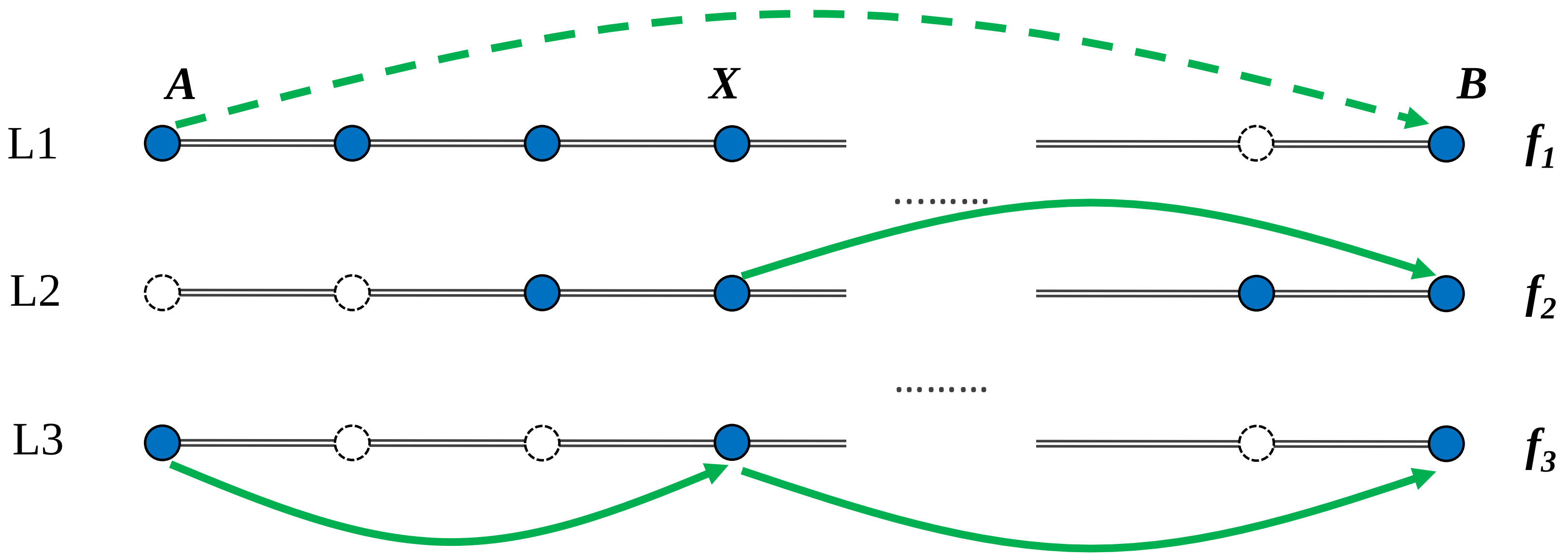
TRIP CHARACTERIZATION

- Itineraries
- Routes



TRIP CHARACTERIZATION

- Itineraries
- Routes
- Hyperpaths (Strategies)



TRANSIT ASSIGNMENT MODELS

Approach	Definition	Authors (year)
Schedule based	Rely on schedules	Liu et al. (2010) Fu et al. (2012)
Frequency based	Don't rely on schedules	

Approach	Definition	Authors (year)
Deterministic	All-or-nothing assignment	Dijkstra (1959) Chriqui & Robillard (1975) Spiess & Florian (1989)
Stochastic	RUM: Split over alternatives	Dial (1971) McFadden (2000)

Approach	Definition	Authors (year)
Dynamic	Sequential	Poon et al. (2004)
Simulation		Cats & Hartl (2016)
Equilibrium	No user can change routes unilaterally	Wardrop (1952) De Cea & Fernández (1993)

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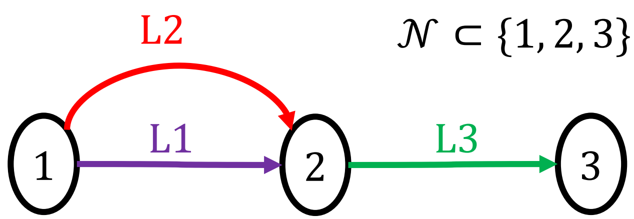
USER EQUILIBRIUM

- Transit networks
  - **No user can change their paths unilaterally**
    - All used paths in the network - same cost – a OD
    - Unused paths must have higher cost



$\mathcal{N} \subset \{1, 2, 3\}$ $\quad \mathcal{L} \subset \{L1, L2, L3\}$

Possible paths

$Path\ 1$: $[(1,2), (L1, L2)]$ → $[(2,3), (L3)]$

$Path\ 2$: $[(1,2), (L1)]$ → $[(2,3), (L3)]$

$Path\ 3$: $[(1,2), (L2)]$ → $[(2,3), (L3)]$

Scenario 1

$T_{1-3} = 5$

Flows	Time
2	5
3	5
0	6

Scenario 2

$T_{1-3} = 50$

Flows	Time
20	10
30	10
0	6

ASSIGNMENT IN CONGESTED SCENARIOS

- Frequency based

	Models	Approach	Authors (year)
Frequency based	Uncongested	Routes (Paths)	Chriqui & Robillard (1975)
	Semi-congested (partially congested: Bouzaïene-Ayari et al., 2001)	Routes (Paths)	De Cea & Fernández (1993)
		Hyperpaths (Strategies)	Wu et al. 1994 Nguyen & Pallottino (1988) Spiess & Florian (1989)
	Fully-congested	Hyperpaths (Strategies)	Cominetti & Correa (2001) Gendreau (1984) Cortés et al. (2013)

DE CEA & FERNANDEZ (1993)

- Contributions
  - Implement congestion effect in boarding process
  - Introduce the concept of effective frequencies
  - Only work based on routes

- Limitations
  - **Semi-congested models**
    - Only waiting time is a function of congestion
    - Flow distribution is a function of nominal frequencies
    - In-vehicle travel time is a function of nominal frequencies
  - **Approximations**
    - Transit users don't consider any subset of services
    - Effective frequencies are only a function of passing through flows
    - Simplify the waiting cost for arcs containing more than one service

NETWORK TOPOLOGY

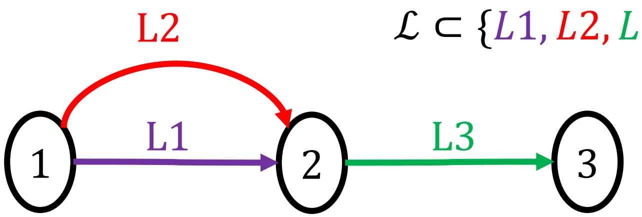
$\mathcal{W} \coloneqq \{w = (i, j) : i, j \in \mathcal{N}, i \neq j\}$

$\mathcal{W} \subset \{(1,2); (1,3); (2,3)\}$

$\mathcal{N} \subset \{1, 2, 3\}$

$\mathcal{L} \subset \{L1, L2, L3\}$

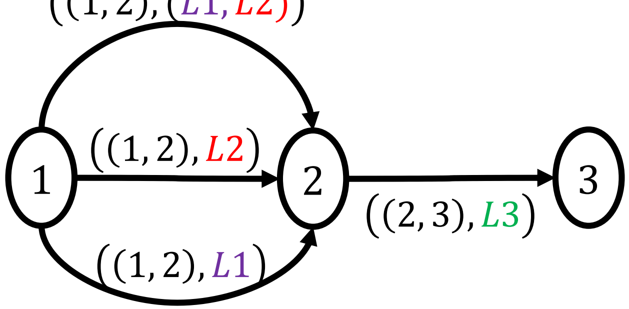
$\mathcal{S} \subseteq \mathcal{W}$ $\quad\quad$ $\mathcal{S} \subset \{(1,2); (2,3)\}$



$\mathcal{L}_s \subseteq \mathcal{L}$

$\mathcal{L}_{(1,2)} \subset \{L1, L2\}$ $\quad$ $\mathcal{L}_{(2,3)} \subset \{L3\}$

$\mathcal{A} \coloneqq \{a = (s_a, L_a) : s_a \in \mathcal{S}, L_a \subseteq \mathcal{L}_{s_a}, L_a \neq \emptyset\}$



$\mathcal{G} = (\mathcal{N}, \mathcal{A})$

FEASIBLE ASSIGNMENT

$$\sum_{a\in\delta_i^-} v_{aw} + T_{wi}^+ = \sum_{a\in\delta_i^+} v_{aw} + T_{wi}^-, \qquad \forall i \in \mathcal{N}, w \in \mathcal{W}$$

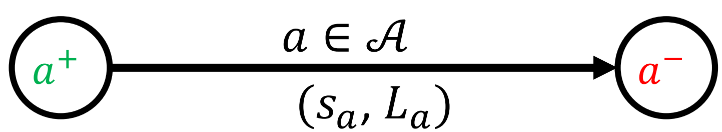
v_{aw} = flow of passengers from w on section s_a using the set of lines $L_a \subseteq \mathcal{L}_{s_a}$

$$T_{wi}^+ = \begin{cases} T_w, & \textit{if } i \textit{ is the origin of } w \\ 0, & \textit{otherwise} \end{cases}$$

$$T_{wi}^- = \begin{cases} T_w, & \textit{if } i \textit{ is the destination of } w \\ 0, & \textit{otherwise} \end{cases}$$

$$\delta_i^+ := \{a \in \mathcal{A} : a^+ = i\}$$

$$\delta_i^- := \{a \in \mathcal{A} : a^- = i\}$$



$$v_a = \sum_{w\in\mathcal{W}} v_{aw}, \qquad \forall a \in \mathcal{A}$$

v_a = total flow of trips on arc a

Ω

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COST FUNCTIONS

w_{il} - expected waiting time at stop i for service l

$$f_{il} \coloneqq \frac{k}{w_{il}}, \qquad \forall i \in \mathcal{N}, l \in L$$

$$w_{iL} \coloneqq \frac{k}{\sum_{l \in L} f_{il}}$$

$$c_a = \frac{\sum_{l \in L_a} t_{s_a l} f_{a^+ l}}{\sum_{l \in L_a} f_{a^+ l}} + \frac{k}{\sum_{l \in L_a} f_{a^+ l}}, \qquad \forall a \in \mathcal{A}$$

$$c_a = \frac{\sum_{l \in L_a} t_{s_a l} f_{a^+ l} + k}{\sum_{l \in L_a} f_{a^+ l}}, \qquad \forall a \in \mathcal{A}$$

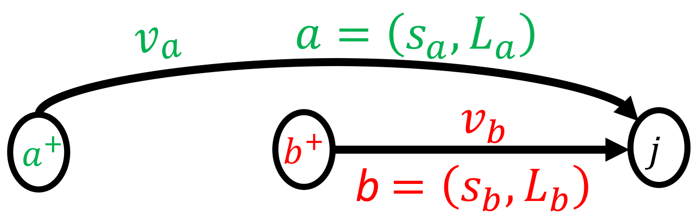
Node a^+ represents the origin node of section s_a

$t_{s_a l}$: In-vehicle travel time on line l in section s_a

KEY CHALLENGES

- Asymmetric cost functions

$$c_a = \frac{\sum_{l \in L_a} t_{s_a l} f_{a^+ l} + k}{\sum_{l \in L_a} f_{a^+ l}}, \qquad \forall a \in \mathcal{A}$$



$$l \in L_a \cap L_b$$

- Diagonalized cost functions

$$\hat{c}_a(v_a) = \frac{\sum_{l \in L_a} t_{s_a l} \hat{f}^a_{a^+ l}(v_a) + k}{\sum_{l \in L_a} \hat{f}^a_{a^+ l}(v_a)}, \qquad \forall a \in \mathcal{A}$$

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EFFECT OF CONGESTION IN WAITING

Number of passengers boarding l in stop i

Number of passengers passing through i in l

Waiting time: $w_{il} = w_{il}(\rho_{il}) = w_{il}\left(\frac{v_{il}^{+} + v_{il}^{\leftrightarrow}}{f_l Q_l}\right)$

Capacity of l

1. $w_{il}(0) = \frac{k}{f_l}$ 2. $\frac{\partial w_{il}(\rho_{il})}{\partial \rho_{il}} \geq 0$

$$f_{il}(\rho_{il}) \coloneqq \frac{k}{w_{il}(\rho_{il})}$$

$$v_{il}^{+} = \sum_{a \in \delta_{il}^{+}} \frac{f_{il}(\rho_{il}) \cdot v_a}{\sum_{l' \in L} f_{il'}(\rho_{il'})}, \qquad \forall i \in \mathcal{N}, l \in \mathcal{L}_i$$

$$v_{il}^{\leftrightarrow} = \sum_{a \in \Delta_{il}} \frac{f_{a^+l}(\rho_{a^+l}) \cdot v_a}{\sum_{l' \in L_a} f_{a^+l'}(\rho_{a^+l'})}, \qquad \forall i \in \mathcal{N}, l \in \mathcal{L}_i$$

EFFECTIVE FREQUENCY & SATURATION OF LINES

$$v_{il}^{+} = \sum_{a \in \delta_{il}^{+}} \frac{f_{il}(\rho_{il}) \cdot v_a}{\sum_{l' \in L} f_{il'}(\rho_{il'})}, \qquad \forall i \in \mathcal{N}, l \in \mathcal{L}_i$$

$$v_{il}^{\leftrightarrow} = \sum_{a \in \Delta_{il}} \frac{f_{a^{+}l}(\rho_{a^{+}l}) \cdot v_a}{\sum_{l' \in L_a} f_{a^{+}l'}(\rho_{a^{+}l'})}, \qquad \forall i \in \mathcal{N}, l \in \mathcal{L}_i$$

$$\rho_{il} = \frac{1}{f_l Q_l}\left[\sum_{a \in \delta_{il}^{+}} \frac{f_{il}(\rho_{il}) \cdot v_a}{\sum_{l' \in L} f_{il'}(\rho_{il'})} + \sum_{a \in \Delta_{il}} \frac{f_{a^{+}l}(\rho_{a^{+}l}) \cdot v_a}{\sum_{l' \in L_a} f_{a^{+}l'}(\rho_{a^{+}l'})}\right], \qquad \forall i \in \mathcal{N}, l \in \mathcal{L}_i$$

$$\rho_{il} = \frac{1}{f_l Q_l}\left[\sum_{a \in \mathcal{C}_{il}} \frac{f_{a^{+}l}(\rho_{a^{+}l}) \cdot v_a}{\sum_{l' \in L_a} f_{a^{+}l'}(\rho_{a^{+}l'})}\right], \qquad \forall i \in \mathcal{N}, l \in \mathcal{L}_i$$

$$\mathcal{C}_{il} \coloneqq \delta_{il}^{+} \cup \Delta_{il}$$

KEY CHALLENGES

- Cost functions are not only an implicit function of flows

$$\rho_{il} = \frac{1}{f_l Q_l}\left[\sum\nolimits_{a \in \mathcal{C}_{il}} \frac{f_{a^+l}(\rho_{a^+l}) \cdot v_a}{\sum_{l' \in L_a} f_{a^+l'}(\rho_{a^+l'})}\right], \qquad \forall i \in \mathcal{N}, l \in \mathcal{L}_i$$

- Fixed point solution

1. Set $n = 0$, $\rho_{il}^0 = 0$.
2. Compute initial effective frequencies, $f_{il}^n \leftarrow k/w_{il}(\rho_{il}^n)$.
3. Compute ρ_{il}^{n+1} using above expression, with f_{il}^n and given flows $\bar{V}$.
4. Iterate until convergence, obtaining $\bar{\rho}_{il}$, and the effective frequencies for the current solution, $\bar{f}_{il}$.

$$\rho_{il} \approx \frac{1}{f_l Q_l}\left[\sum\nolimits_{a \in \mathcal{C}_{il}} \frac{\bar{f}_{a^+l} \cdot \bar{v}_a}{\sum_{l' \in L_a} \bar{f}_{a^+l'}}\right], \qquad \forall i \in \mathcal{N}, l \in \mathcal{L}_i$$

$$\hat{c}_a(v_a) = \frac{\sum_{l \in L_a} t_{s_a l} \hat{f}_{a^+l}^a(v_a) + k}{\sum_{l \in L_a} \hat{f}_{a^+l}^a(v_a)}, \qquad \forall a \in \mathcal{A}$$

KEY CHALLENGES

- Cost functions are not only an implicit function of flows

$$\rho_{il} = \frac{1}{f_l Q_l}\left[\sum\nolimits_{a\in\mathcal{C}_{il}} \frac{f_{a^+l}(\rho_{a^+l})\cdot v_a}{\sum_{l'\in L_a} f_{a^+l'}(\rho_{a^+l'})}\right], \qquad \forall i \in \mathcal{N}, l \in \mathcal{L}_i$$

$$\hat{\rho}_{il}^{a}(v_a) \approx \frac{1}{f_l Q_l}\left[\sum\nolimits_{b\in\mathcal{C}_{il}} \frac{\bar{f}_{b^+l}}{\sum_{l'\in L_b} \bar{f}_{b^+l'}}\cdot\left((1-\Delta_b^a)\bar{v}_b + \Delta_b^a v_b\right)\right], \qquad \forall i \in \mathcal{N}, l \in \mathcal{L}_i$$

$$\Delta_b^a = \begin{cases} 1, & if\ a = b, \\ 0, & otherwise \end{cases}$$

$$\rho_{il} \approx \frac{1}{f_l Q_l}\left[\sum\nolimits_{a\in\mathcal{C}_{il}} \frac{\bar{f}_{a^+l}\cdot \bar{v}_a}{\sum_{l'\in L_a} \bar{f}_{a^+l'}}\right], \qquad \forall i \in \mathcal{N}, l \in \mathcal{L}_i$$

$$\hat{c}_a(v_a) = \frac{\sum_{l\in L_a} t_{s_a l}\hat{f}_{a^+l}^{a}(v_a) + k}{\sum_{l\in L_a} \hat{f}_{a^+l}^{a}(v_a)}, \qquad \forall a \in \mathcal{A}$$

KEY CHALLENGES

- Asymmetric cost functions
- Cost functions are not only an implicit function of flows
- Number of arcs are combinatorial

$$\text{Number of arcs} = 2^{|\mathcal{L}_s|} - 1$$

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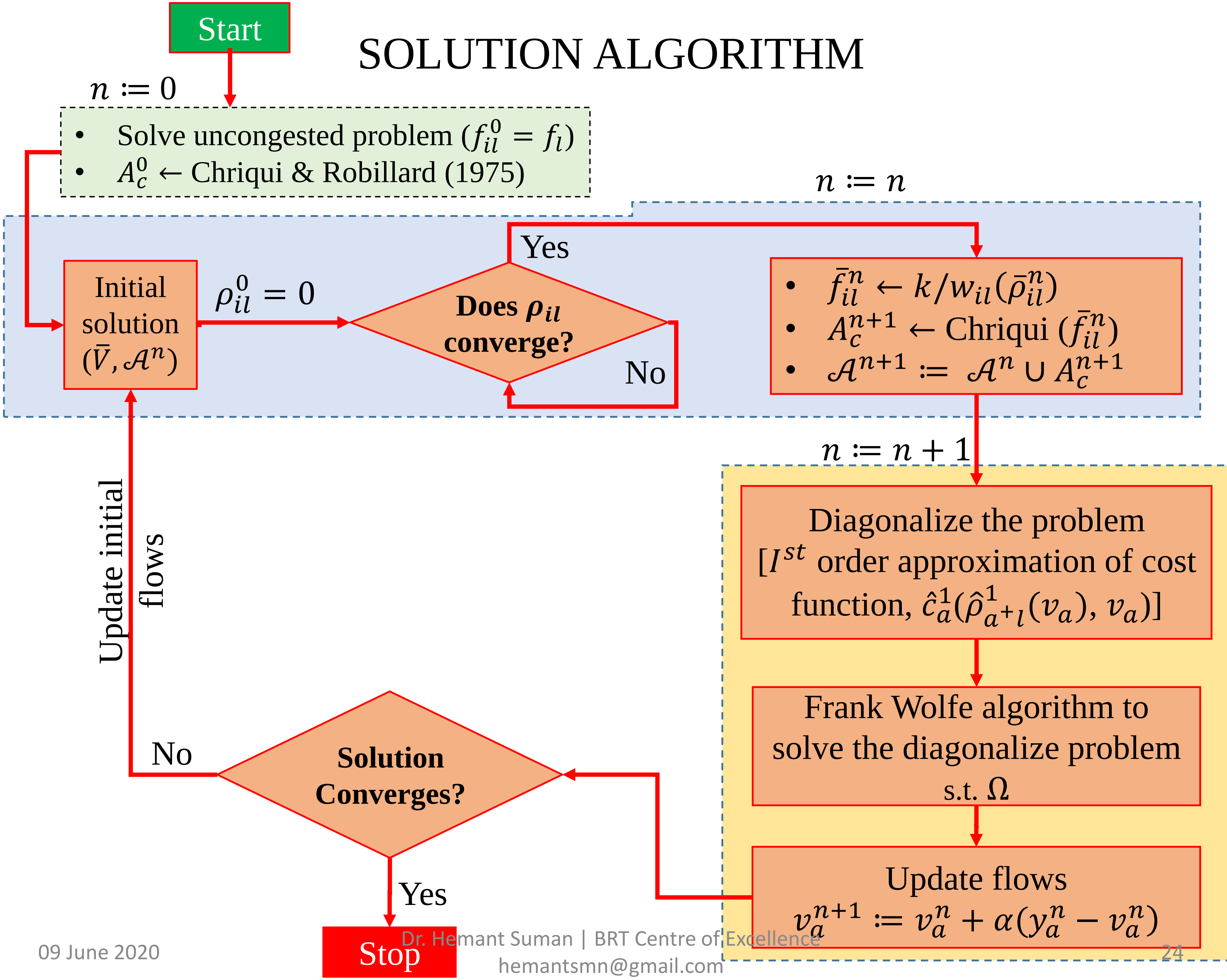
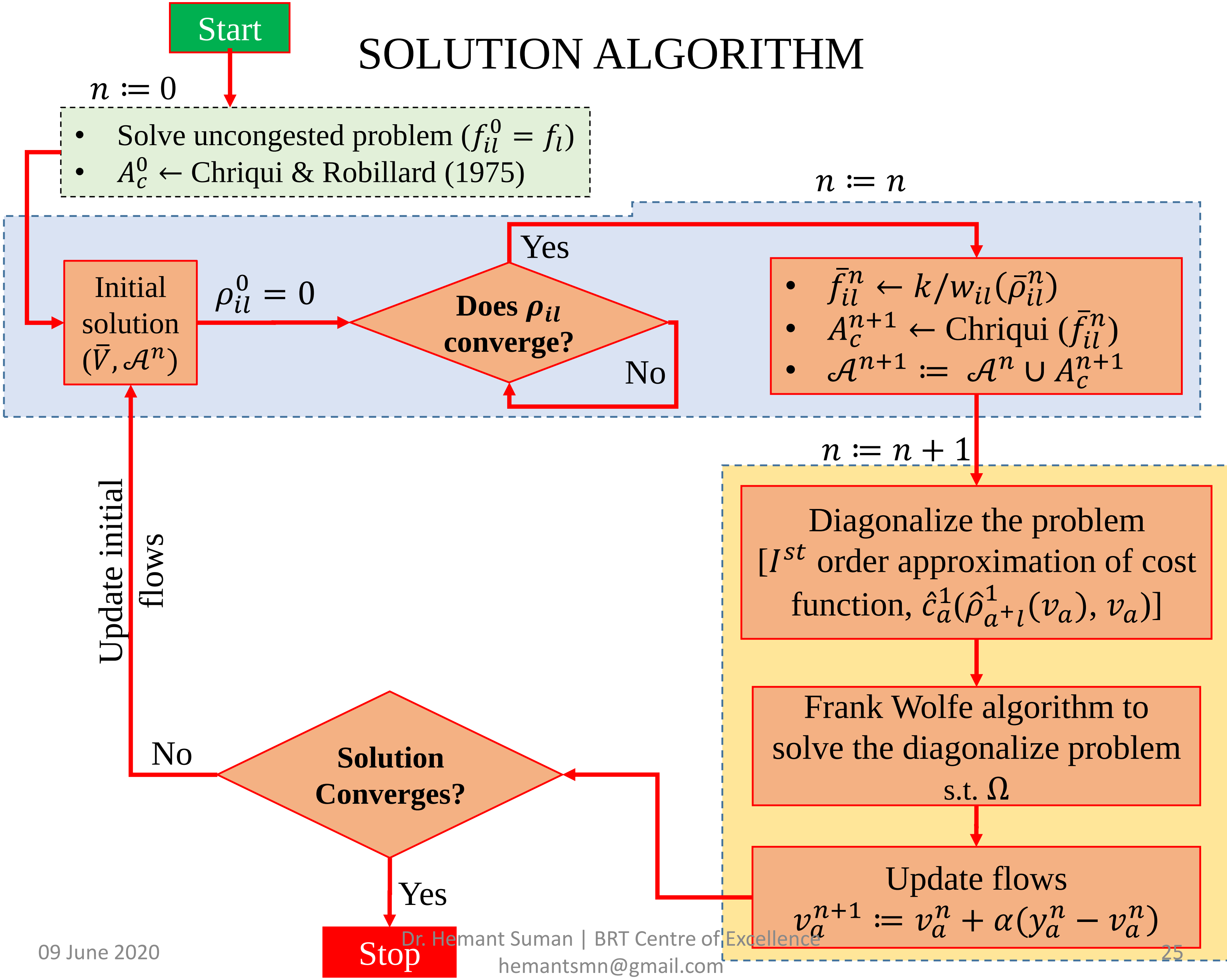
SOLUTION ALGORITHM



SOLUTION ALGORITHM



IMPLEMENTATION

- BPR function
  - Waiting time

$$w_{il} \coloneqq \frac{k}{f_l} + \beta\left(\frac{v_{il}^{+} + v_{il}^{\leftrightarrow}}{f_l Q_l}\right)^n, \qquad \forall i \in \mathcal{N}, l \subseteq \mathcal{L}$$

  - Effective frequencies

$$f_{il} = k / w_{il}$$

$$f_{il} = f_l \left[\frac{1}{1 + f_l\left(\frac{\beta}{k}\right)\left(\frac{v_{il}^{+} + v_{il}^{\leftrightarrow}}{f_l Q_l}\right)^n}\right]$$

IMPLEMENTATION

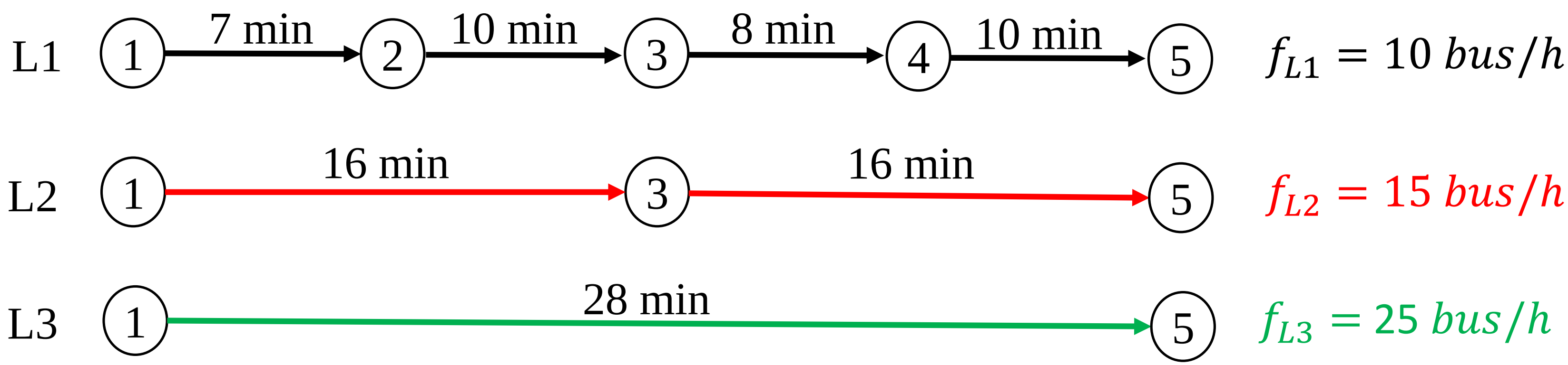
- BPR function

	This work	De Cea & Fernandez (1993)
Effective frequencies (f_{il})	$f_l\left[\frac{1}{1+f_l\left(\frac{\beta}{k}\right)\left(\frac{v_{il}^{+}+v_{il}^{\leftrightarrow}}{f_lQ_l}\right)^n}\right]$	$f_l\left[\frac{1}{1+f_l\left(\frac{\beta}{k}\right)\left(\frac{v_{il}^{\leftrightarrow}}{f_lQ_l}\right)^n}\right]$
Flow split ($v_{il}^{+}/v_{il}^{\leftrightarrow}$)	Using effective frequencies (f_{il})	Using nominal frequencies (f_l)
Cost function (c_a)	$\frac{\sum_{l\in L_a} t_{s_a l} f_{a^+l}}{\sum_{l\in L_a} f_{a^+l}}+\frac{k}{\sum_{l\in L_a} f_{a^+l}}$	$\frac{\sum_{l\in L_a} t_{s_a l} f_l}{\sum_{l\in L_a} f_l}+\frac{k}{\sum_{l\in L_a} f_l}+\beta\left(\frac{v_{a^+l}^{+}+v_{a^+l}^{\leftrightarrow}}{f_lQ_l}\right)^n$

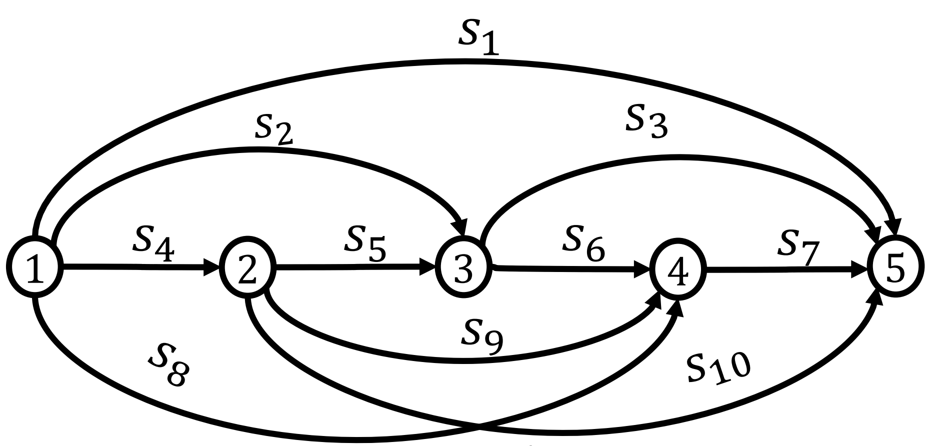
In-vehicle time — Waiting time (This work)

In-vehicle time — Waiting time (De Cea & Fernandez (1993))

5-STOP TRANSIT NETWORK



O-D (w)	1-2	1-3	1-4	1-5	2-3	2-4	2-5	3-4	3-5	4-5
T_w	0	0	0	350	0	0	100	0	80	70



$(\mathcal{N}, S)$

5-STOP TRANSIT NETWORK

- This work's solution

Sections (s)	Set of services ($\mathcal{L}_s$)	Set of relevant arcs ($\mathcal{A}$)
s_1	$L1, L2, L3$	
s_2	$L1, L2$	
s_3	$L1, L2$	
s_4	$L1$	
s_5	$L1$	
s_6	$L1$	
s_7	$L1$	
s_8	$L1$	
s_9	$L1$	
s_{10}	$L1$	

5-STOP TRANSIT NETWORK

- This work's solution

$n := 0$

Sections (s)	Set of services ($\mathcal{L}_s$)	Set of relevant arcs ($\mathcal{A}$)	Uncongested Solution	
			v_a	c_a
s_1	$L1, L2, L3$	$\{s_1, L3\}$	–	30.4
			–	–
			–	–
s_2	$L1, L2$	$\{s_2, (L1, L2)\}$	–	18.8
s_3	$L1, L2$	$\{s_3, (L1, L2)\}$	–	19.2
s_4	$L1$	$\{s_4, L1\}$	–	13.0
s_5	$L1$	$\{s_5, L1\}$	–	16.0
s_6	$L1$	$\{s_6, L1\}$	–	14.0
s_7	$L1$	$\{s_7, L1\}$	–	16.0
s_8	$L1$	$\{s_8, L1\}$	–	31.0
s_9	$L1$	$\{s_9, L1\}$	–	24.0
s_{10}	$L1$	$\{s_{10}, L1\}$	–	34.0

Uncongested Solution

- Nominal frequencies
- Chriqui's algorithm

5-STOP TRANSIT NETWORK

- This work's solution

Sections (s)	Set of services ($\mathcal{L}_s$)	Set of relevant arcs ($\mathcal{A}$)	$n := 0$ Uncongested Solution v_a	c_a	$n := n$ 0th Iteration v_a	c_a
s_1	$L1, L2, L3$	$\{s_1, L3\}$	–	30.4	350.0	44.4
		$\{s_1, (L2, L3)\}$	–	–	–	34.4
			–	–	–	–
s_2	$L1, L2$	$\{s_2, (L1, L2)\}$	–	18.8	0.0	18.8
s_3	$L1, L2$	$\{s_3, (L1, L2)\}$	–	19.2	80.0	22.0
s_4	$L1$	$\{s_4, L1\}$	–	13.0	0.0	13.0
s_5	$L1$	$\{s_5, L1\}$	–	16.0	0.0	26.0
s_6	$L1$	$\{s_6, L1\}$	–	14.0	0.0	26.4
s_7	$L1$	$\{s_7, L1\}$	–	16.0	70.0	35.4
s_8	$L1$	$\{s_8, L1\}$	–	31.0	0.0	31.0
s_9	$L1$	$\{s_9, L1\}$	–	24.0	0.0	34.0
s_{10}	$L1$	$\{s_{10}, L1\}$	–	34.0	100.0	44.0

<u>0th Iteration</u>

- Effective frequencies
- Chriqui's algorithm

5-STOP TRANSIT NETWORK

- This work's solution

			$n \coloneqq 0$		$n \coloneqq n$		$n \coloneqq n+1$	
Sections (s)	Set of services ($\mathcal{L}_s$)	Set of relevant arcs ($\mathcal{A}$)	Uncongested Solution		0th Iteration		1st Iteration	
			v_a	c_a	v_a	c_a	v_a	c_a
s_1	$L1, L2, L3$	$\{s_1, L3\}$	–	30.4	350.0	44.4	184.3	40.7
		$\{s_1, (L2, L3)\}$	–	–	–	34.4	165.7	35.9
		$\{s_1, (L1, L2, L3)\}$	–	–	–	–	–	35.4
s_2	$L1, L2$	$\{s_2, (L1, L2)\}$	–	18.8	0.0	18.8	0.0	20.4
s_3	$L1, L2$	$\{s_3, (L1, L2)\}$	–	19.2	80.0	22.0	80.0	24.7
s_4	$L1$	$\{s_4, L1\}$	–	13.0	0.0	13.0	0.0	13.0
s_5	$L1$	$\{s_5, L1\}$	–	16.0	0.0	26.0	0.0	26.0
s_6	$L1$	$\{s_6, L1\}$	–	14.0	0.0	26.4	0.0	27.3
s_7	$L1$	$\{s_7, L1\}$	–	16.0	70.0	35.4	70.0	36.3
s_8	$L1$	$\{s_8, L1\}$	–	31.0	0.0	31.0	0.0	31.0
s_9	$L1$	$\{s_9, L1\}$	–	24.0	0.0	34.0	0.0	34.0
s_{10}	$L1$	$\{s_{10}, L1\}$	–	34.0	100.0	44.0	100.0	44.0

1st Iteration

- Solve diagonalized problem
- New effective frequencies
- Chriqui's algorithm

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5-STOP TRANSIT NETWORK

- This work's solution

$n := 0$ $n := n$ $n := n + 1$

Sections (s)	Set of services ($\mathcal{L}_s$)	Set of relevant arcs ($\mathcal{A}$)	Uncongested Solution		0th Iteration		1st Iteration			Final Iteration	
			v_a	c_a	v_a	c_a	v_a	c_a		v_a	c_a
s_1	$L1, L2, L3$	$\{s_1, L3\}$	–	30.4	350.0	44.4	184.3	40.7		0.0	36.6
		$\{s_1, (L2, L3)\}$	–	–	–	34.4	165.7	35.9		45.0	34.7
		$\{s_1, (L1, L2, L3)\}$	–	–	–	–	–	35.4		305.0	34.7
s_2	$L1, L2$	$\{s_2, (L1, L2)\}$	–	18.8	0.0	18.8	0.0	20.4		0.0	22.8
s_3	$L1, L2$	$\{s_3, (L1, L2)\}$	–	19.2	80.0	22.0	80.0	24.7	……	80.0	26.4
s_4	$L1$	$\{s_4, L1\}$	–	13.0	0.0	13.0	0.0	13.0		0.0	21.0
s_5	$L1$	$\{s_5, L1\}$	–	16.0	0.0	26.0	0.0	26.0		0.0	34.0
s_6	$L1$	$\{s_6, L1\}$	–	14.0	0.0	26.4	0.0	27.3		0.0	34.9
s_7	$L1$	$\{s_7, L1\}$	–	16.0	70.0	35.4	70.0	36.3		70.0	43.9
s_8	$L1$	$\{s_8, L1\}$	–	31.0	0.0	31.0	0.0	31.0		0.0	39.0
s_9	$L1$	$\{s_9, L1\}$	–	24.0	0.0	34.0	0.0	34.0		0.0	42.0
s_{10}	$L1$	$\{s_{10}, L1\}$	–	34.0	100.0	44.0	100.0	44.0		100.0	52.0

5-STOP TRANSIT NETWORK

- De Cea & Fernandez (1993)

Sections (s)	Set of services ($\mathcal{L}_s$)	Set of relevant arcs ($\mathcal{A}$)
s_1	$L1, L2, L3$	
s_2	$L1, L2$	
s_3	$L1, L2$	
s_4	$L1$	
s_5	$L1$	
s_6	$L1$	
s_7	$L1$	
s_8	$L1$	
s_9	$L1$	
s_{10}	$L1$	

5-STOP TRANSIT NETWORK

- De Cea & Fernandez (1993)

$n := 0$

Sections (s)	Set of services ($\mathcal{L}_s$)	Set of relevant arcs ($\mathcal{A}$)	Uncongested Solution	
			v_a	c_a
s_1	$L1, L2, L3$	$\{s_1, L3\}$	–	30.4
			–	–
s_2	$L1, L2$	$\{s_2, (L1, L2)\}$	–	18.8
s_3	$L1, L2$	$\{s_3, (L1, L2)\}$	–	19.2
s_4	$L1$	$\{s_4, L1\}$	–	13.0
s_5	$L1$	$\{s_5, L1\}$	–	16.0
s_6	$L1$	$\{s_6, L1\}$	–	14.0
s_7	$L1$	$\{s_7, L1\}$	–	16.0
s_8	$L1$	$\{s_8, L1\}$	–	31.0
s_9	$L1$	$\{s_9, L1\}$	–	24.0
s_{10}	$L1$	$\{s_{10}, L1\}$	–	34.0

<u>Uncongested Solution</u>

- Nominal frequencies
- Chriqui's algorithm

5-STOP TRANSIT NETWORK

- De Cea & Fernandez (1993)

$n := 0$ $n := n$

Sections (s)	Set of services ($\mathcal{L}_s$)	Set of relevant arcs ($\mathcal{A}$)	Uncongested Solution		0th Iteration	
			v_a	c_a	v_a	c_a
s_1	$L1, L2, L3$	$\{s_1, L3\}$	–	30.4	350.0	44.4
		$\{s_1, (L1, L2)\}$	–	–	–	35.6
s_2	$L1, L2$	$\{s_2, (L1, L2)\}$	–	18.8	0.0	18.8
s_3	$L1, L2$	$\{s_3, (L1, L2)\}$	–	19.2	80.0	22.0
s_4	$L1$	$\{s_4, L1\}$	–	13.0	0.0	13.0
s_5	$L1$	$\{s_5, L1\}$	–	16.0	0.0	26.0
s_6	$L1$	$\{s_6, L1\}$	–	14.0	0.0	26.4
s_7	$L1$	$\{s_7, L1\}$	–	16.0	70.0	35.4
s_8	$L1$	$\{s_8, L1\}$	–	31.0	0.0	31.0
s_9	$L1$	$\{s_9, L1\}$	–	24.0	0.0	34.0
s_{10}	$L1$	$\{s_{10}, L1\}$	–	34.0	100.0	44.0

0th Iteration

- Nominal frequencies
- Chriqui's algorithm excluding $L3$
- Find disjoint set of services

5-STOP TRANSIT NETWORK

- De Cea & Fernandez (1993)

$n := 0$ $n := n$ $n := n + 1$

Sections (s)	Set of services ($\mathcal{L}_s$)	Set of relevant arcs ($\mathcal{A}$)	Uncongested Solution		0th Iteration		1st Iteration			Final Iteration	
			v_a	c_a	v_a	c_a	v_a	c_a		v_a	c_a
s_1	$L1, L2, L3$	$\{s_1, L3\}$	–	30.4	350.0	44.4	206.9	38.7		205.9	38.6
		$\{s_1, (L1, L2)\}$	–	–	–	35.6	143.1	38.6		144.1	38.6
s_2	$L1, L2$	$\{s_2, (L1, L2)\}$	–	18.8	0.0	18.8	0.0	21.7		0.0	21.8
s_3	$L1, L2$	$\{s_3, (L1, L2)\}$	–	19.2	80.0	22.0	80.0	25.2		80.0	25.2
s_4	$L1$	$\{s_4, L1\}$	–	13.0	0.0	13.0	0.0	19.2	……	0.0	19.3
s_5	$L1$	$\{s_5, L1\}$	–	16.0	0.0	26.0	0.0	32.2		0.0	32.3
s_6	$L1$	$\{s_6, L1\}$	–	14.0	0.0	26.4	0.0	32.9		0.0	33.0
s_7	$L1$	$\{s_7, L1\}$	–	16.0	70.0	35.4	70.0	41.9		70.0	42.0
s_8	$L1$	$\{s_8, L1\}$	–	31.0	0.0	31.0	0.0	37.2		0.0	37.3
s_9	$L1$	$\{s_9, L1\}$	–	24.0	0.0	34.0	0.0	40.2		0.0	40.3
s_{10}	$L1$	$\{s_{10}, L1\}$	–	34.0	100.0	44.0	100.0	50.2		100.0	50.3

5-STOP TRANSIT NETWORK

- Cost

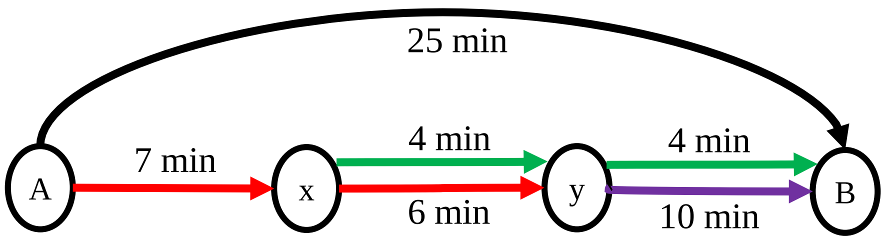
			This work					De Cea & Fernandez (1993)				
Sections (s)	Set of services ($\mathcal{L}_s$)	Set of relevant arcs ($\mathcal{A}$)	Solution		Flow distribution			Solution		Flow distribution		
			v_a	c_a	v_{L1}	v_{L2}	v_{L3}	v_a	c_a	v_{L1}	v_{L2}	v_{L3}
s_1	$L1, L2, L3$	$\{s_1, (L2, L3)\}$	45.0	**34.7**								
		$\{s_1, (L1, L2, L3)\}$	305.0	**34.7**								
		$\{s_1, L3\}$						205.9	**38.6**			
		$\{s_1, (L1, L2)\}$						144.1	**38.6**			

5-STOP TRANSIT NETWORK

- Flow distribution

			This work					De Cea & Fernandez (1993)				
Sections (s)	Set of services $(\mathcal{L}_s)$	Set of relevant arcs $(\mathcal{A})$	Solution		Flow distribution			Solution		Flow distribution		
			v_a	c_a	v_{L1}	v_{L2}	v_{L3}	v_a	c_a	v_{L1}	v_{L2}	v_{L3}
s_1	$L1, L2, L3$	$\{s_1, (L2, L3)\}$	**45.0**	34.7		19.0	26.0					
		$\{s_1, (L1, L2, L3)\}$	**305.0**	34.7	80.0	96.0	129.0					
		$\{s_1, L3\}$						**206**	38.6			205.9
		$\{s_1, (L1, L2)\}$						**144**	38.6	62.6	81.5	
		Total Flows			**80.0**	**115.0**	**155.0**			**62.6**	**81.5**	**205.9**
		Total Flows			**279.0**	**166.0**	**155.0**			**260**	**134**	**205.9**

4-STOP TRANSIT NETWORK



$f_{L1} = 10\ bus/h$

$f_{L2} = 10\ bus/h$

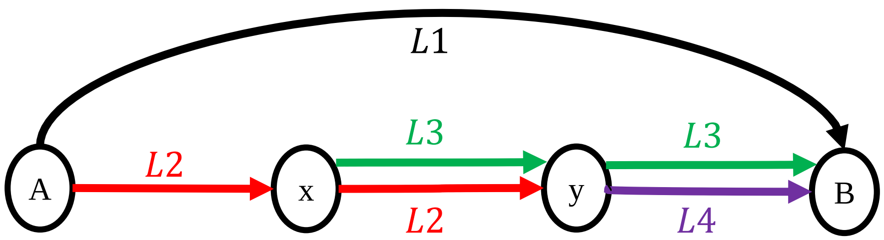
$f_{L3} = 4\ bus/h$

$f_{L4} = 20\ bus/h$

Transit network example (Spiess & Florian, 1989; De Cea & Fernandez, 1993)

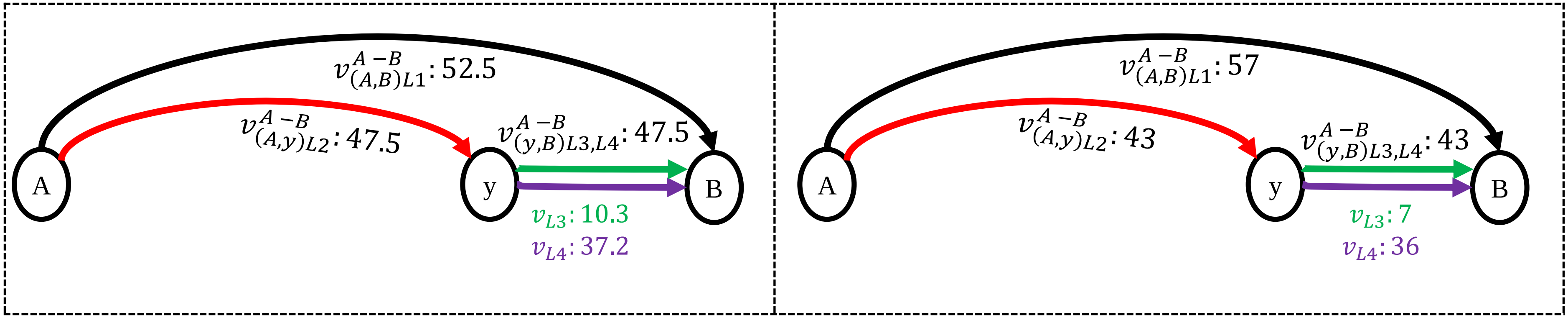
4-STOP TRANSIT NETWORK

- Mild Congestion ($\boldsymbol{n = 1, \beta = 10}$) $T_{A-B} = 100$



This work

De Cea & Fernandez (1993)



4-STOP TRANSIT NETWORK

- Mild Congestion $(\boldsymbol{n = 1, \beta = 10})$ $T_{A-B} = 100$

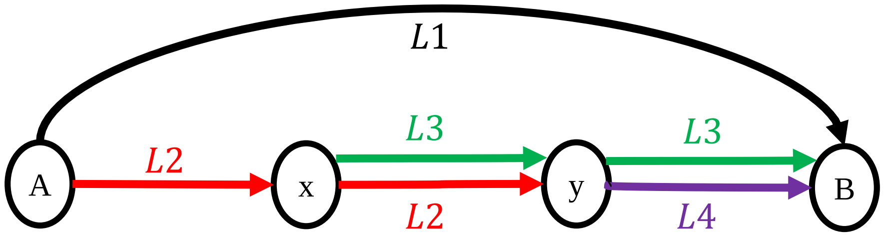
Set of relevant arcs $(\mathcal{A})$	This work			De Cea & Fernandez (1993)				
	v_a	c_a	c_p	v_a	c_a	c_p	c'_a	c'_p
$\{A-B(L1)\}$	52.5	36.3	36.3	57.0	36.7	36.7	36.7	36.7
$\{A-y(L2)\}$	47.5	23.8	36.3	43.0	23.3	35.7	23.3	36.6
$\{y-B(L3,L4)\}$	47.5	12.5		43.0	12.4		13.3	
$\{A-x(L2)\}$	0.0	17.7	40.7	0.0	17.3	40.3	17.3	40.3
$\{x-B(L3)\}$	0.0	23.0		0.0	23.0		23.0	
$\{x-y(L2,L3)\}$	0.0	11.4		0.0	11.3		12.8	

c_a & c_p: Estimated using **THIS WORK's** cost functions

c'_a & c'_p: Estimated using **De Cea & Fernandez** cost functions

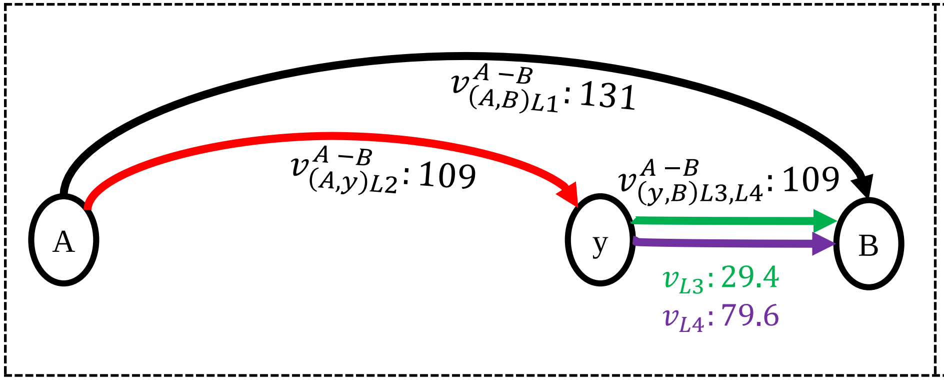
4-STOP TRANSIT NETWORK

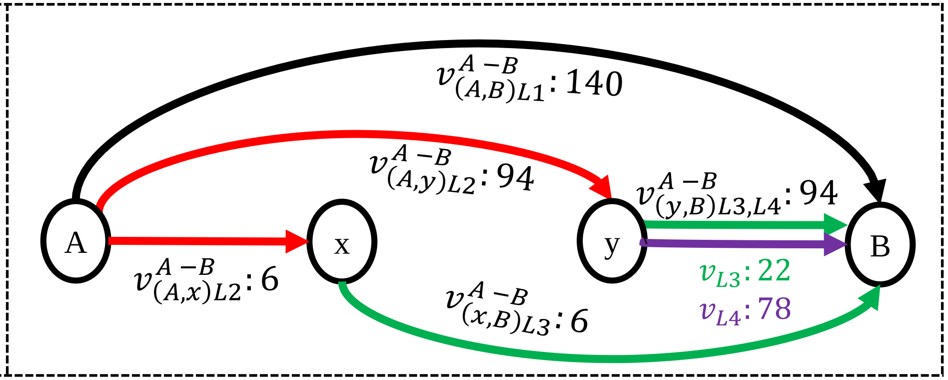
- High Congestion ($n = 1, \beta = 20$) $T_{A-B} = 240$



This work

De Cea & Fernandez (1993)





CONCLUSIONS

- **Fully-congested models**
  - Waiting time
  - Flow distribution
  - In-vehicle travel time

  (Waiting time, flow distribution and in-vehicle travel time together: f_{il})

- **Addresses three key challenges**
  - Asymmetric cost function
  - Cost functions – not only implicit - V
  - Combinatorial

- **Implications of the work**
  - Disjoint sets of services – yield higher cost
  - Semi-congested models: not in equilibrium

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¡Muchas gracias!

Bueno, eso concluye mi presentación de hoy. Muchas gracias por su amable atención. Ahora preguntas por favor pero solo en inglés.

¿Preguntas?

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